# *A Sparse Blind Source Separation Based Method for Harmonic Emission Levels Assessment*

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***Abstract:*** Harmonic emission levels assessment is significant for harmonic mitigation. Traditional assessment methods are easy to be impacted by the background harmonics. Besides, when the harmonic sources are correlative to each other, the calculation errors increase. To solve this problem, a sparse blind source separation based method is applicated in this paper. This method can obtain accurate calculation results even when the background harmonics fluctuate greatly and the harmonic sources are related to each other. Simulation analysis verifies the validity of the proposed method.

## 1. Introduction

Recently, with the widely connecting of power electronics devices, a large number of complex harmonic sources appear into the power grid, which aggravates the harmonic pollution of the power system. To mitigate this harmonic problem, it is necessary to assess the harmonic emission levels of each harmonic source at the point of common coupling (PCC)[1,2,3].

Based on the standard IEC61000-3-6, the key to assess the harmonic emissions is calculating the harmonic impedance of the utility side[3]. The existing calculating methods include the fluctuation method[1][2], the regression method[4][5], the covariance characteristic method[3], and the fast independent component analysis method (FastICA)[6,7,8], etc. Among these algorithms, the fluctuation method and the regression method are only suitable for scenarios where the background harmonics are stable. The covariance characteristic method is based on the assumption that the harmonic impedance of the customer side is far larger than that of the utility side, thus the harmonic current at the PCC and the background harmonic are only weak related. This method can release the impacts of the background harmonics on the calculating process, however, when the background harmonics fluctuate greatly, the calculating results still have large errors.

Among the existing methods, FastICA based method assumes that the harmonic sources of both sides are independent from each other, thus, have a strong ability of reducing the impacts from background harmonics fluctuating. However, in application, this assumption is becoming more and more difficult to establish since many harmonic sources are pulse width modulation (PWM) based, and so, the harmonic sources of both sides are related in a certain degree. In these cases, the signal separation ability of FastICA is not satisfactory and the calculated harmonic impedances are with large errors.

To overcome the shortcomings of the existing methods above, sparse blind source separating methods (SBSS)[9,10,11,12] is applied in this paper. SBSS, depending on the spare properties of source signals, is a novel technology in blind source separation field and is gradually applied in various engineering fields. In this paper, we construct the blind source separation mode for the harmonic emission levels assessment. And then, SBSS is used to solve this problem. Based on the wavelet packet transform[13,14], the source signals can be transformed into its sparse form. After that, the observed signals will be clustered linearly. It can be proved mathematically that the corresponding slopes of these clustering line are equal to the harmonic impedances. Furthermore, we can assess the harmonic emissions based on these calculated impedances. Simulation results indicate that, compared with the existing algorithms, SBSS method can assess the harmonic emissions more precisely. Besides, the relevance between the source signals on the both sides does not affect the accuracy of this method.

## 2. The Mode of Harmonic Emission Level Assessment

The power grid can be separated into the utility side and the customer side from the PCC. The corresponding Norton equivalent circuit is shown in Figure 1.

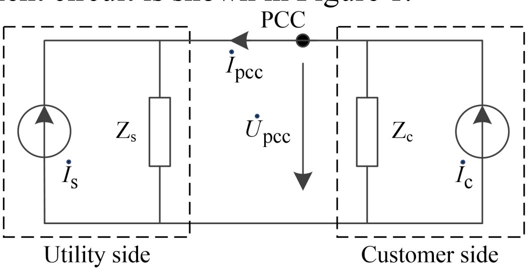



Figure 1: The Norton equivalent circuit of harmonic emissions.

According to the superposition principle and the Figure 1, we have

$$\begin{bmatrix} \dot{U}_{pcc} \\ \dot{I}_{pcc} \end{bmatrix} = \begin{bmatrix} \frac{Z_s Z_c}{Z_s + Z_c} & \frac{Z_s Z_c}{Z_s + Z_c} \\ -\frac{Z_s}{Z_s + Z_c} & \frac{Z_c}{Z_s + Z_c} \end{bmatrix} \begin{bmatrix} \dot{I}_s \\ \dot{I}_c \end{bmatrix} \tag{1}$$

Where $\dot{U}_{pcc}$ and $\dot{I}_{pcc}$ are the harmonic voltages and currents measured at the PCC, respectively. Zs and Zc are the harmonic impedances of the utility and customers sides, respectively. $\dot{I}_s$ and $\dot{I}_c$ are the harmonic sources of both sides, respectively.

According to the standard IEC61000-3-6, the harmonic emission levels of the customer and the utility sides can be calculated from Eq. (2).

$$\begin{cases} \dot{U}_{pcc\text{-}c} = \dot{I}_{pcc} Z_s \\ \dot{U}_{pcc\text{-}s} = \dot{U}_{pcc} - \dot{U}_{pcc\text{-}c} \end{cases} \tag{2}$$

Eq. (2) shows that, the key to assess the harmonic emissions of both sides is calculating Zs.

## 3. Sparse Blind Source Separation Technology

To solve the assessment problem above, in this section, the blind source separation mode and SBSS technology are introduced in detail.

### 3.1. The Blind Source Separation Mode of Harmonic Emission Levels Assessment

The blind source separation mode of Eq. (1) can be presented as

$$\begin{bmatrix} U_{\text{pcc-x}} \\ U_{\text{pcc-y}} \\ I_{\text{pcc-x}} \\ I_{\text{pcc-y}} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} I_{\text{c-x}} \\ I_{\text{c-y}} \\ I_{\text{s-x}} \\ I_{\text{s-y}} \end{bmatrix} \tag{3}$$

Where $U_{\text{pcc-x}}$ 、 $U_{\text{pcc-y}}$ 、 $I_{\text{pcc-x}}$ and $I_{\text{pcc-y}}$ present the real and imaginary parts of $\dot{U}_{\text{pcc}}$ and $\dot{I}_{\text{pcc}}$, respectively, while $I_{\text{c-x}}$ 、 $I_{\text{c-y}}$ 、 $I_{\text{s-x}}$ and $I_{\text{s-y}}$ present the real and imaginary parts of $\dot{I}_{\text{s}}$ and $\dot{I}_{\text{c}}$, respectively. The element $a_{ij}$ （ $i,j$=1,2,3,4 ） is the mixing coefficient, which is related to the harmonic impedances of both sides.

Eq. (4) can be represented as

$$\boldsymbol{X} = \boldsymbol{AS} \tag{4}$$

Where $X=[U_{pcc-x}, U_{pcc-y}, I_{pcc-x}, I_{pcc-y}]^T$ are the observed signals, and $S=[I_{c-x}, I_{c-y}, I_{s-x}, I_{-y}]^T$ are the source signals.

If we find an appropriate blind source separation method, the source signals can be separated from the observed signals one by one. The separated signals **Y** correspond to $[Ic\text{-}x, Ic\text{-}y, Is\text{-}x, Is\text{-}y]^{\text{T}}$, but with the ordering indeterminacies [6,7,8]. Further, we have $\boldsymbol{X} = \hat{\boldsymbol{A}}\boldsymbol{Y}$, where $\hat{\boldsymbol{A}}$ can be calculated by the least square method as:

$$\hat{\boldsymbol{A}} = \boldsymbol{XY}^{\text{T}}\left(\boldsymbol{YY}^{\text{T}}\right)^{-1} \tag{5}$$

Therefore, according to $\hat{\boldsymbol{A}}$, the utility harmonic impedance can be calculated. Based on the facts that the real part of impedance is always positive, the ordering indeterminacies in the blind source separating method can be solved. For any column of the matrix **A**, we have

$$\text{Z}_{\text{x}} = \frac{\hat{a}_{1j}\hat{a}_{3j} + \hat{a}_{2j}\hat{a}_{4j}}{\hat{a}_{3j}^{\ 2} + \hat{a}_{4j}^{\ 2}} = \begin{cases} \text{Z}_{\text{s-x}}, \text{if } \text{Z}_{\text{x}} > 0 \\ \text{Z}_{\text{c-x}}, \text{if } \text{Z}_{\text{x}} < 0 \end{cases} \tag{6}$$

Where Zs-x and Zc-x are the real parts of Zs and Zc, respectively.

Further, by using the same column of matrix **A**, we can calculate the imaginary part of the corresponding harmonic impedance as:

$$Z_y = \frac{\hat{a}_{2j}\hat{a}_{3j} - \hat{a}_{1j}\hat{a}_{4j}}{\hat{a}_{3j}^{\ 2} + \hat{a}_{4j}^{\ 2}} \tag{7}$$

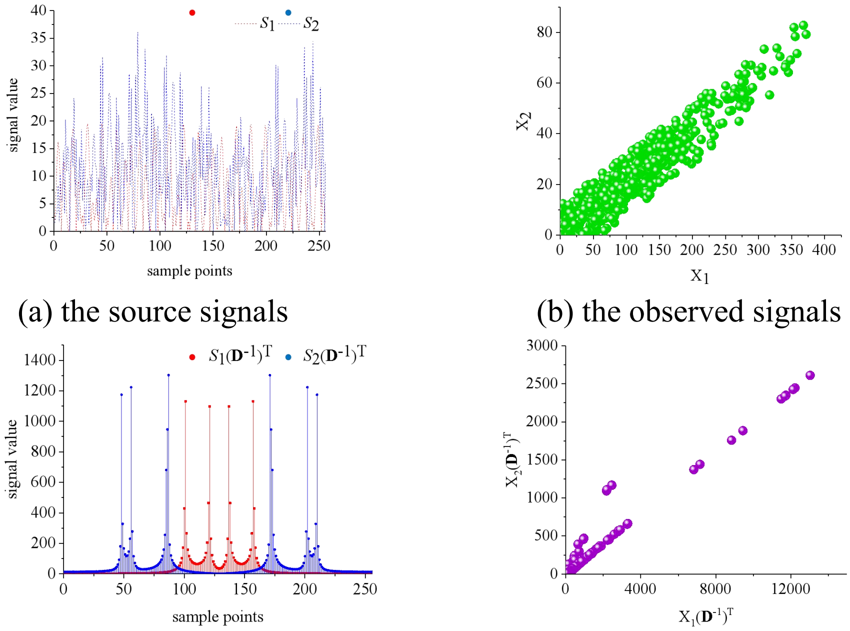
Consequently, Zs is obtained and we can further assess the harmonic emission levels.

### 3.2. The Theory of Sparse Blind Source Separation Technology

The above analysis is based on the blind source separation mode. To solve this mode, an appropriate method is needed. Among the related algorithms, FastICA, as a classical method in this researching field, receives widely applications. This method is based on the assumption that the source signals at both sides of the PCC is nearly independent. This assumption is established for traditional nonlinear customer. However, for harmonic sources with PWM, the source signals of both sides are related in a certain degree. Therefore, in these situations, the signal separation ability of FastICA is unsatisfactory.

To solve this problem, SBSS, a novel blind source separation method, is used in this paper. This method uses the sparse properties of signals to separate the sources and without the assumption that the sources are independent from each other. The basic theory of SBSS is as the following.

In blind source separation mode, if the source signals are sparse, they are zero mostly and non-zero in a few times. Therefore, it can be considered that the observed signals are generated from at most one source at any time. In this situation, the observed signals will cluster linearly as is shown in Figure 2. Through the slopes of these lines, the matrix $\boldsymbol{A}$ can be calculated[9,10,11]. Figure 2 also shows that, for situations that sources are not sparse, the observed signals are generating from all the sources, thus, there is no obvious clustering phenomenon.




(a) the source signals (b) the observed signals

(c) the sparse source signals (d) the observed signals with sparse sources

Figure 2: The theory of SBSS method.

In practice, the source signals $\boldsymbol{S}$ are not sparse and a sparse dictionary $\mathbf{D}$ is needed for SBSS method. Through this sparse dictionary $\mathbf{D}$, the sparse form of the source signals can be obtained as $\boldsymbol{S}(\mathbf{D}^{-1})^{\mathrm{T}}$. Therefore, we can transform Eq. (4) into

$$X(\mathbf{D}^{-1})^{T} = AS(\mathbf{D}^{-1})^{T} \quad (8)$$

Eq. (8) indicates that acting **D** on $X$ is equivalent to acting **D** on $S$, and this transformation won't change the matrix $A$. Therefore, the harmonic impedances calculated from Eq. (8) and Eq. (4) are the same.

The sparse dictionary **D** can be calculated based on the wavelet packet transform [13,14]. To simplify notation, we define $S^{D}=S(D^{-1})^{T}$ and $X^{D}=X(D^{-1})^{T}$. If the transformed source signals $S^{D}$ are sparse, the corresponding observed signals $X^{D}$ are generated from at most one source in most times. For instance, if the value of $I_{c\text{-}x}{}^{D}$, $I_{c\text{-}y}{}^{D}$, and $I_{s\text{-}x}{}^{D}$ is zero, $X^{D}$ is generated only from $I_{s\text{-}y}{}^{D}$. Therefore, we have

$$\begin{bmatrix} U_{\text{pcc-x}}{}^{D} \\ U_{\text{pcc-y}}{}^{D} \\ I_{\text{pcc-x}}{}^{D} \\ I_{\text{pcc-y}}{}^{D} \end{bmatrix} = \begin{bmatrix} a_{14} I_{\text{s-y}}{}^{D} \\ a_{24} I_{\text{s-y}}{}^{D} \\ a_{34} I_{\text{s-y}}{}^{D} \\ a_{44} I_{\text{s-y}}{}^{D} \end{bmatrix} \quad (9)$$

And so,

$$\begin{cases} U_{\text{pcc-x}}{}^{D} / U_{\text{pcc-y}}{}^{D} = a_{14} / a_{24} \\ U_{\text{pcc-x}}{}^{D} / I_{\text{pcc-x}}{}^{D} = a_{14} / a_{34} \\ U_{\text{pcc-x}}{}^{D} / I_{\text{pcc-y}}{}^{D} = a_{14} / a_{44} \end{cases} \quad (10)$$

In this case, $X^{D}$ is linear clustering, and based on the corresponding slopes, we can calculate the elements in column 4 of the matrix $A$. Similarly, if any three source signals in Eq. (8) are sparse, $X^{D}$ can also cluster into lines. With the slopes of these lines, we can calculate the matrix $A$, and further, the harmonic impedance of utility sides is obtained.

## 4. Simulation Analysis

To compare the effects of SBSS method with other existing methods, based on the Norton equivalent circuit in Figure 1 and Eq.(1), the simulation data are generated as the following.

(1) harmonic sources:

The amplitude and angle of $\dot{I}_c$ are set as 100 A and -30°, respectively. The amplitude of $\dot{I}_s$ is $k$ times that of $\dot{I}_c$ , and the angle of $\dot{I}_s$ is 30°. To reflect the impacts of the background harmonics, we set $k$=0.6, 0.8, 1.0, 1.2, 1.4, 1.6 in this paper. Additionally, ±5% random fluctuations are added into the amplitude and angle of both $\dot{I}_c$ and $\dot{I}_s$ , respectively. Besides, by considering that the harmonic source signals of both sides are correlated to each other in some scenarios, 10% sinusoidal fluctuations are added into the amplitude and angle of both $\dot{I}_c$ and $\dot{I}_s$, respectively.

(2) harmonic impedances:

We set Zs=3+13j Ω and Zc=15+60j Ω. Meanwhile, ±10% sinusoidal fluctuations are added into the real and imaginary parts of both Zs and Zc, respectively.

8000 simulation data are generated based on the above principle, and four methods, i.e.(1) the regression method,(2) the covariance characteristic method,(3) FastICA, and(4) SBSS are used to calculate the utility harmonic impedance. The errors of these methods are shown in Figure 3.

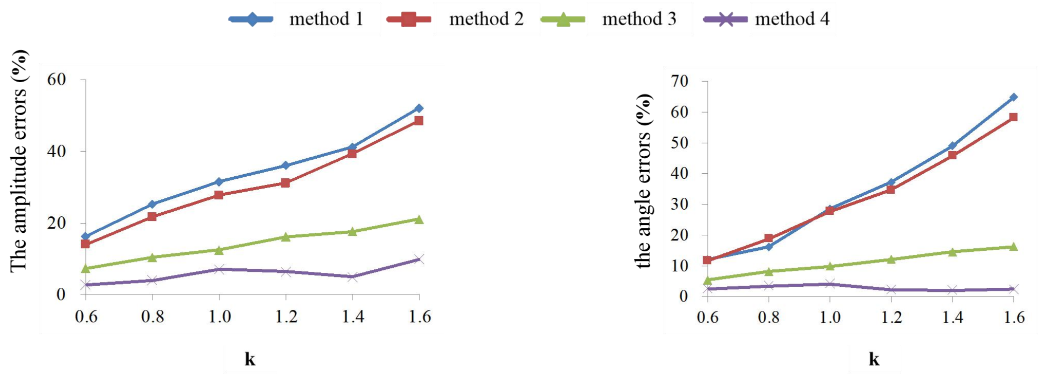



Figure 3: The errors of Zs calculated from the four methods.

It is indicated in Figure 3 that the errors of the regression method and the covariance characteristic method are always high, and with the increasing of k, their errors increase further. This is because the results for the both methods are easy to be impacted by the background harmonics. For FastICA method, the ability of defending the impacts from the background harmonics is improved. However, since sinusoidal fluctuations are added into the both source signals, the necessary assumption of this method that the sources are independent from each other is not hold. Therefore, the calculation errors of FastICA is still large. Compared with these traditional algorithms, the SBSS method can ensure the calculation accuracy.

The effects of the SBSS method can be further presented by enhancing the correlation between the source signals of both sides. This correlation can be assessed through Eq. (11).

$$\rho_{sc} = \frac{Cov\left(\left|\dot{I}_S\right|,\left|\dot{I}_C\right|\right)}{\sqrt{D\left(\left|\dot{I}_S\right|\right)}\sqrt{D\left(\left|\dot{I}_C\right|\right)}} \tag{11}$$

Where $Cov\left(\left|\dot{I}_S\right|,\left|\dot{I}_C\right|\right)$ presents the covariance between $\left|\dot{I}_S\right|$ and $\left|\dot{I}_C\right|$, and $D(*)$ presents calculating the variance.

Figure 4 shows that with the enhancing of the sinusoidal fluctuation amplitudes, the correlation between $\dot{I}_c$ and $\dot{I}_s$ are increased.

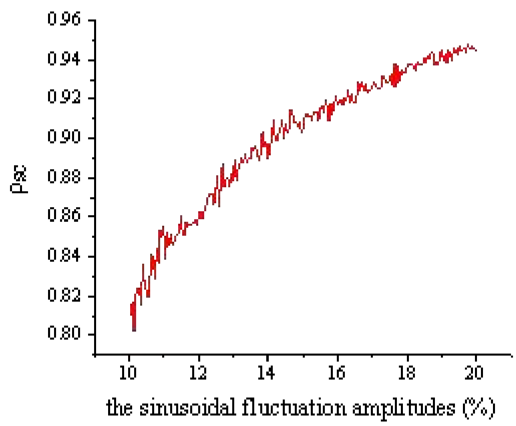



Figure 4: The relationship between the sinusoidal fluctuation amplitudes and *ρsc*.

By setting $k$=1 and changing the sinusoidal fluctuation amplitudes, the calculation errors of FastICA and SBSS methods are shown in Figure 5. It is indicated that, with the increasing for the correlation between $\dot{I}_c$ and $\dot{I}_s$, the calculation errors of FastICA increases. By comparison, the errors of SBSS is always low.

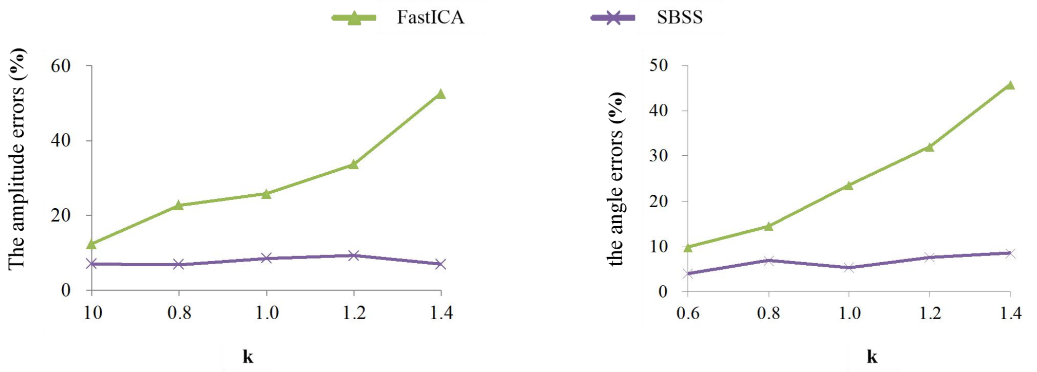



Figure 5: The calculation errors of FastICA and SBSS.

Consequently, the SBSS method can calculate the utility harmonic impedance accurately, even when the source signals are relative to each other. Further, it can assess the harmonic emissions correctly.

## 5. Conclusions

A sparse blind source separation based method for harmonic emission levels assessment is proposed in this paper. Through the wavelet packet transform and the decomposing of the real and imaginary parts for the harmonic source, the source signals can be sparse. And then, the corresponding measured harmonic voltage and current data will cluster linearly. Based on the slopes of these clustering line, the utility harmonic impedance can be calculated. Further, the harmonic emission levels can be assessed. This blind source separation based method doesn't need the assumption that the harmonic sources at both sides of the PCC are independent from each other. Besides, simulation analysis results indicate that, this method can guarantee a high accuracy of calculation.

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